Name: Key

Date: _____ Period: _____

Unit 6: Sequences

Notes: Geometric Sequences

Main Ideas/Questions	Notes
Geometric Sequences	A sequence in which the pattern of the sequence is being multiplied
Common Ratio (fraction)	$\frac{2nd\ term}{1st\ term}, \frac{3rd\ term}{2nd\ term}, \frac{4th\ term}{3rd\ term}$ = common ratio (must all equal the same #)
Identifying a Geometric Sequence	**Determine whether the following represent geometric sequences. If yes, identify the common ratio.**

1. 2, 10, 50, 250, ...

$\frac{10}{2}=5$ $\frac{250}{50}=5$ $\frac{50}{10}=5$ yes $r=5$

2. 135, 45, 15, 5, ...

$\frac{45}{135}=\frac{1}{3}$ $\frac{5}{15}=\frac{1}{3}$ $\frac{15}{45}=\frac{1}{3}$ yes $r=\frac{1}{3}$

3. 6, 18, 24, 30, ...

$\frac{18}{6}=3$ $\frac{24}{18}=\frac{4}{3}$ no

4. 7, -14, 28, -56, ...

$\frac{-14}{7}=-2$ $\frac{-56}{28}=-2$ $\frac{28}{-14}=-2$ yes $r=-2$

5. 80, -40, 20, -10, ...

$\frac{-40}{80}=-\frac{1}{2}$ $\frac{-10}{20}=-\frac{1}{2}$ $\frac{20}{-40}=-\frac{1}{2}$ yes, $r=-\frac{1}{2}$

6. -9, -36, -144, -576, ...

$\frac{-36}{-9}=4$ $\frac{-576}{-144}=4$ $\frac{-144}{-36}=4$ yes, $r=4$

Continuing Geometric Sequences	**Given the geometric sequence, find the next three terms.**

7. 7, -21, 63, -189, 567, -1701

×-3 ×-3 ×-3

$\frac{-21}{7}=-3=r$

8. 3072, 768, 192, 48, 12, 3

$\frac{768}{3072}=\frac{1}{4}=r$

9. 8, 4, 2, 1, 1/2, 1/4

$\frac{4}{8}=\frac{1}{2}=r$

10. -5, -25, -125, -625, -3125, -15625

$\frac{-25}{-5}=5=r$

Geometric Sequence Formula	**The n^{th} term of a geometric sequence can be found using the following formula:**

$$a_n = a_1(r)^{n-1}$$

where a_1 = 1st term on the list and r = common ratio

$$a_n = a_1(r)^{n-1}$$

Examples

Write the rule for the n^{th} term, then find a_7.

11. 3, 9, 27, ...

$a_1 = 3$

$\frac{9}{3} = 3 = r$

$a_n = a_1(r)^{n-1}$

$a_n = 3(3)^{n-1}$

$a_7 = 3(3)^{7-1}$

$a_7 = 3(3)^6$

$a_7 = 2187$

12. -4, 20, -100, ...

$a_1 = -4$

$\frac{20}{-4} = -5 = r$

$a_n = a_1(r)^{n-1}$

$a_n = -4(-5)^{n-1}$

$a_7 = -4(-5)^{7-1}$

$a_7 = -4(-5)^6$

$a_7 = -62{,}500$

13. 400, 200, 100, ...

$a_1 = 400$

$\frac{200}{400} = \frac{1}{2} = r$

$a_n = 400(\frac{1}{2})^{n-1}$

$a_7 = 400(\frac{1}{2})^{7-1}$

$a_7 = 400(\frac{1}{2})^6$

$a_7 = \frac{25}{4}$

14. 1, 5, 25, ...

$a_1 = 1$

$\frac{5}{1} = 5 = r$

$a_n = 1(5)^{n-1}$

$a_7 = 1(5)^{7-1}$

$a_7 = 1(5)^6$

$a_7 = 15{,}625$

15. -1, -4, -16, ...

$a_1 = -1$

$\frac{-4}{-1} = 4 = r$

$a_n = -1(4)^{n-1}$

$*a_5 = -1(4)^{5-1}$

$a_5 = -1(4)^4$

$a_5 = -256$

$a_7 = -1(4)^{7-1}$

$a_7 = -1(4)^6$

$a_7 = -4096$

16. 729, -243, 81, ...

$a_1 = 729$

$\frac{-243}{729} = -\frac{1}{3} = r$

$a_n = 729(-\frac{1}{3})^{n-1}$

$*a_4 = 729(-\frac{1}{3})^{4-1}$

$a_4 = 729(-\frac{1}{3})^3$

$a_4 = -27$

$a_7 = 729(-\frac{1}{3})^{7-1}$

$a_7 = 729(-\frac{1}{3})^6$

$a_7 = 1$

17. 6, -12, 24, ...

$a_1 = 6$

$\frac{-12}{6} = -2 = r$

$a_n = 6(-2)^{n-1}$

$*a_{12} = 6(-2)^{12-1}$

$a_{12} = 6(-2)^{11}$

$a_{12} = -12{,}288$

$a_7 = 6(-2)^{7-1}$

$a_7 = 6(-2)^6$

$a_7 = 384$

18. 8, 12, 18, ...

$a_1 = 8$

$\frac{12}{8} = \frac{3}{2} = r$

$a_n = 8(\frac{3}{2})^{n-1}$

$a_7 = 8(\frac{3}{2})^{7-1}$

$a_7 = 8(\frac{3}{2})^6$

$a_7 = \frac{729}{8}$

Application

Year	Value ($)
1	10,000
2	8,000
3	6,400

The table to the left shows a car's value for 3 years after it is purchased.

19. Write a rule to represent the car's depreciation.

$a_1 = 10{,}000$

$\frac{8000}{10000} = \frac{4}{5} = r$

$a_n = 10{,}000(\frac{4}{5})^{n-1}$

20. What will be the value of the car after 10 years?

$a_{10} = 10{,}000(\frac{4}{5})^{10-1}$

$a_{10} = 10{,}000(\frac{4}{5})^9$

$a_{10} = \$1342.18$