

Name: Key  
 Date: \_\_\_\_\_ Period: \_\_\_\_\_

Unit 6: Sequences  
 Notes: Arithmetic Sequences

Main Ideas/Questions	Notes/Examples																
<b>Arithmetic Sequence</b>	A sequence in which the pattern is being added (+) or subtracted (-) by the same amount between all numbers																
<b>Common Difference</b>	2 <sup>nd</sup> number - 1 <sup>st</sup> number is the <i>common difference</i>																
	Must check for each term																
	Ex. 3 <sup>rd</sup> number - 2 <sup>nd</sup> number, 4 <sup>th</sup> number - 3 <sup>rd</sup> number, etc.																
<b>Identifying an Arithmetic Sequence</b>	<b>Determine whether the sequences are arithmetic. If yes, identify the common difference.</b>																
	<table border="0"> <tr> <td>1. 1, 5, 9, 13, ...</td> <td>5-1=4</td> <td>2. 1, 3, 5, 8, ...</td> <td>3-1=2</td> </tr> <tr> <td>yes</td> <td>9-5=4</td> <td>no</td> <td>5-3=2</td> </tr> <tr> <td></td> <td>13-9=4</td> <td></td> <td>8-5=3</td> </tr> </table>	1. 1, 5, 9, 13, ...	5-1=4	2. 1, 3, 5, 8, ...	3-1=2	yes	9-5=4	no	5-3=2		13-9=4		8-5=3				
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<table border="0"> <tr> <td>3. 8, 6, 4, 2, ...</td> <td>6-8=-2</td> <td>4. -5, -8, -11, -14, ...</td> <td>-8-(-5)=-3</td> </tr> <tr> <td>yes</td> <td>4-6=-2</td> <td>yes</td> <td>-11-(-8)=-3</td> </tr> <tr> <td></td> <td>2-4=-2</td> <td></td> <td>-14-(-11)=-3</td> </tr> </table>	3. 8, 6, 4, 2, ...	6-8=-2	4. -5, -8, -11, -14, ...	-8-(-5)=-3	yes	4-6=-2	yes	-11-(-8)=-3		2-4=-2		-14-(-11)=-3					
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<table border="0"> <tr> <td>5. 5, 10, 20, 40, ...</td> <td>10-5=5</td> <td>6. 7, 6, 5, 4, ...</td> <td>6-7=-1</td> </tr> <tr> <td>no</td> <td>20-10=10</td> <td>yes</td> <td>5-6=-1</td> </tr> <tr> <td></td> <td></td> <td></td> <td>4-5=-1</td> </tr> </table>	5. 5, 10, 20, 40, ...	10-5=5	6. 7, 6, 5, 4, ...	6-7=-1	no	20-10=10	yes	5-6=-1				4-5=-1					
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<b>Continuing an Arithmetic Sequence</b>	<b>Given the arithmetic sequence, find the next three terms.</b>																
	<table border="0"> <tr> <td>7. 9, 13, 17, 21, <u>25</u>, <u>29</u>, <u>33</u></td> <td>13-9=4</td> </tr> <tr> <td>+4 +4 +4 +4 +4 +4</td> <td>↑</td> </tr> <tr> <td></td> <td>common difference</td> </tr> </table>	7. 9, 13, 17, 21, <u>25</u> , <u>29</u> , <u>33</u>	13-9=4	+4 +4 +4 +4 +4 +4	↑		common difference										
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<table border="0"> <tr> <td>8. 5, 2, -1, -4, <u>-7</u>, <u>-10</u>, <u>-13</u></td> <td>2-5=-3</td> </tr> <tr> <td>-3 -3 -3 -3 -3</td> <td>↑</td> </tr> <tr> <td></td> <td>common difference</td> </tr> </table>	8. 5, 2, -1, -4, <u>-7</u> , <u>-10</u> , <u>-13</u>	2-5=-3	-3 -3 -3 -3 -3	↑		common difference											
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<table border="0"> <tr> <td>9. -8, -2, 4, 10, <u>16</u>, <u>22</u>, <u>28</u></td> <td>-2-(-8)=6</td> </tr> <tr> <td>+6 +6 +6 +6 +6</td> <td>↑</td> </tr> <tr> <td></td> <td>common difference</td> </tr> </table>	9. -8, -2, 4, 10, <u>16</u> , <u>22</u> , <u>28</u>	-2-(-8)=6	+6 +6 +6 +6 +6	↑		common difference											
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<b>Arithmetic Sequence Formula</b>	<p>The <math>n^{\text{th}}</math> term of an arithmetic sequence can be found by using the following formula:</p> $a_n = a_1 + d(n - 1)$ <p>where: <math>n</math> = the number of the term you are looking for, <math>d</math> = the common difference, and <math>a_1</math> = the 1<sup>st</sup> number you see in the sequence given</p>																
<b>Examples</b>	<b>Write the rule for the <math>n^{\text{th}}</math> term, then find <math>a_{19}</math>.</b>																
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	<p>12. -11, -8, -5, -2, ...</p> $a_1 = -11 \quad d = -8 - (-11) = 3$ $a_n = -11 + 3(n-1)$ $a_n = -11 + 3n - 3$ $a_n = 3n - 14$ $a_{19} = 3(19) - 14 = 43$	<p>13. -2, 0, 2, 4, ...</p> $a_1 = -2 \quad d = 0 - (-2) = 2$ $a_n = -2 + 2(n-1)$ $a_n = -2 + 2n - 2$ $a_n = 2n - 4$ $a_{19} = 2(19) - 4 = 34$
	<p>14. -16, -21, -26, -31, ...</p> $a_1 = -16 \quad d = -21 - (-16) = -5$ $a_n = -16 + (-5)(n-1)$ $a_n = -16 - 5n + 5$ $a_n = -5n - 11$ $a_{19} = -5(19) - 11 = -106$	<p>15. 101, 92, 83, 74, ...</p> $a_1 = 101 \quad d = 92 - 101 = -9$ $a_n = 101 + (-9)(n-1)$ $a_n = 101 - 9n + 9$ $a_n = -9n + 110$ $a_{19} = -9(19) + 110 = -61$
<p><b>Applications</b></p>	<p>16. You visit the Grand Canyon and drop a penny off the edge of the cliff. The distance the penny will fall is 16 feet for the first second, 48 feet for the next second, 80 feet the third second, and so on.</p> <p style="text-align: center;">16, 48, 80, ...</p> <p>a. Write a formula to represent this sequence.</p> $a_1 = 16 \quad a_n = 16 + 32(n-1)$ $d = 48 - 16 = 32 \quad a_n = 16 + 32n - 32$ $a_n = 32n - 16$ <p>b. How far will the penny have traveled after 6 seconds?</p> $a_6 = 32(6) - 16$ $a_6 = 176 \text{ ft.}$	
	<p>17. The total bank loan for Sarah's new car is \$15,265. The bank automatically withdraws \$295.80 each month to pay off the car.</p> $d = -295.80 \quad a_1 = 15,265$ <p>a. Write a formula to represent this sequence.</p> $a_n = 15,265 + (-295.80)(n-1)$ $a_n = 15,265 - 295.80n + 295.80$ $a_n = -295.80n + 15,560.80$ <p>b. What will be the balance of the loan after 2 years?</p> <p>2 years = 24 months = n</p> $a_{24} = -295.80(24) + 15,560.80$ $a_{24} = \$8461.60$	